

Recoilless resonant neutrino capture and basics of neutrino oscillations

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Abstract

It is shown that the experiment on recoilless resonant emission and absorption of $\bar{\nu}_e$, proposed recently by Raghavan, could have an important impact on our understanding of the physics of neutrino oscillations.

1 Introduction

Evidence for neutrino oscillations obtained in the Super-Kamiokande atmospheric [1], SNO solar [2], KamLAND reactor[3] and other neutrino experiments [4, 5, 6, 7] is one of the most important recent discoveries in particle physics. There is no natural explanation of the smallness of neutrino masses and of the large mixing angles in the Standard Model (SM). There is a general opinion that small neutrino masses and neutrino mixing are signatures of new physics beyond the SM.

All existing neutrino oscillation data with the exception of the LSND data [8])¹ are in a good agreement with three-neutrino mixing. In the framework of three-neutrino mixing, from the analysis of the Super-Kamiokande atmospheric neutrino data the following ranges for the largest neutrino mass-squared difference Δm_{23}^2 and for the mixing angle were obtained [1]²

$$1.5 \cdot 10^{-3} \leq \Delta m_{23}^2 \leq 3.4 \cdot 10^{-3} \text{eV}^2; \quad \sin^2 2\theta_{23} > 0.92 \quad (1)$$

¹Indication in favor of $\bar{\nu}_\mu \rightleftharpoons \bar{\nu}_e$ oscillations obtained in the accelerator short-baseline LSND experiment are going to be checked by the running MiniBooNE experiment [9].

² Neutrino mass-squared difference is given by $\Delta m_{ik}^2 = m_k^2 - m_i^2$.

From a global analysis of the KamLAND and solar neutrino data was found [3]

$$\Delta m_{12}^2 = 7.9^{+0.6}_{-0.5} 10^{-5} \text{ eV}^2; \quad \tan^2 \theta_{12} = 0.40^{+0.10}_{-0.07}. \quad (2)$$

The investigation of neutrino oscillations is based on the following assumptions:

1. Neutrino interactions are the SM charged current (CC) and neutral current (NC) interactions. The leptonic CC and neutrino NC are given by

$$j_{\alpha}^{\text{CC}}(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x) \gamma_{\alpha} l_L(x); \quad j_{\alpha}^{\text{NC}}(x) = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x) \gamma_{\alpha} \nu_{lL}(x) \quad (3)$$

2. The fields of neutrinos with definite masses enter into CC and NC in *the mixed form*

$$\nu_{lL}(x) = \sum_{k=1}^3 U_{lk} \nu_{kL}(x). \quad (4)$$

Here $\nu_k(x)$ is the field of neutrino with mass m_k and U is the unitary PMNS matrix [10, 11]

In the case of neutrino mixing (4), the flavor lepton numbers L_e , L_{μ} and L_{τ} are not conserved. For the three-neutrino mixing the standard probability of the transition $\nu_l \rightarrow \nu_{l'}$ is given by (see [12])

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \sum_{k=1}^3 U_{l'k} e^{-i\Delta m_{1k}^2 \frac{L}{2E}} U_{lk}^* \right|^2, \quad (5)$$

where L is the distance between the neutrino-detection and neutrino-production points and E is the neutrino energy. Taking into account the unitarity of the mixing matrix we can rewrite (5) in the following form

$$P(\nu_l \rightarrow \nu_{l'}) = |\delta_{l'l} + \sum_{k \neq 1} U_{l'k} (e^{-i\Delta m_{1k}^2 \frac{L}{2E}} - 1) U_{lk}^*|^2 \quad (6)$$

The probabilities $P(\nu_l \rightarrow \nu_{l'})$ in general depend on six parameters. However, because of the smallness of the parameters $\frac{\Delta m_{12}^2}{\Delta m_{23}^2}$ and $\sin^2 \theta_{13}$ in the leading approximation neutrino oscillations in the atmospheric-LBL and solar-KamLAND regions are described by the simplest two-neutrino expressions

which depend, correspondingly, on Δm_{23}^2 , $\sin^2 2\theta_{23}$ and Δm_{12}^2 , $\tan^2 \theta_{12}$ (see review [12]). The numerical values of these parameters were obtained from the analysis of the experimental data by using two-neutrino expressions.

Several derivations of Eq. (5), which are based on different physical assumptions, exist in the literature. The aim of this paper is to propose a way of testing these assumptions. We will show that an experiment using recoilless resonant antineutrino emission and capture, proposed recently [13, 14], could provide such a possibility.

2 Different approaches to neutrino oscillations

We discuss here different points of view on the physics of neutrino oscillations.

Neutrinos are produced in CC weak processes. For the difference of momenta of neutrinos with masses m_k and m_i (in the rest-frame of the source) we have

$$\Delta p_{ik} = (p_k - p_i) \sim \frac{\Delta m_{ik}^2}{E}, \quad (7)$$

where E is the neutrino energy (in standard neutrino oscillation experiments $E \gtrsim \text{MeV}$). From (7), (1), and (2) follows that $|\Delta p_{ik}|$ is much smaller than the quantum-mechanical uncertainty of the momentum. Thus, it is impossible to distinguish the emission of neutrinos with different masses in neutrino-production processes. The matrix element of the production of ν_k together with lepton l^+ in a process $a \rightarrow b + l^+ + \nu_k$ (a and b are some hadrons) has the form [15]

$$\langle l^+ \nu_k b | S | a \rangle \simeq U_{lk}^* \langle l^+ \nu_l b | S | a \rangle_{SM}, \quad (8)$$

where $\langle l^+ \nu_l b | S | a \rangle_{SM}$ is the Standard Model matrix element of the process

$$a \rightarrow b + l^+ + \nu_l \quad (9)$$

in which neutrino masses can be safely neglected.

From (9) follows:

1. The state of the flavor neutrino ν_l which is produced in a CC weak process together with l^+ is given by

$$|\nu_l\rangle = \sum_k U_{lk}^* |\nu_k\rangle, \quad (10)$$

where $|\nu_k\rangle$ is the state of a neutrino with mass m_k and 4-momentum $p_k = E_k, \vec{p}_k$.

2. The probabilities of the processes of neutrino production are given by the SM.

Thus, in a charged-current weak process a flavor neutrino ν_l , which is described by the state (10), is produced. What will be the state of the neutrino after some time t (at some distance L)? Two different approaches to the propagation of neutrino states are discussed in the literature.

I. Evolution in time.

The evolution equation of any quantum system is the Schrödinger equation (see, for example, [16])

$$i \frac{\partial |\Psi(t)\rangle}{\partial t} = H |\Psi(t)\rangle. \quad (11)$$

Here $|\Psi(t)\rangle$ is the state of the system at the time t and H is the total Hamiltonian. The general solution of this equation has the form

$$|\Psi(t)\rangle = e^{-iHt} |\Psi(0)\rangle, \quad (12)$$

where $|\Psi(0)\rangle$ is the state of the system at the initial time $t = 0$.

If $|\Psi(0)\rangle = |\nu_l\rangle$, we have for the neutrino state in vacuum at the time t

$$|\nu_l\rangle_t = e^{-iHt} |\nu_l\rangle = \sum_k e^{-iE_k t} U_{lk}^* |\nu_k\rangle. \quad (13)$$

Thus, if the energies E_k are different, the neutrino state $|\nu_l\rangle_t$ is a *non-stationary* one. For such states the time-energy uncertainty relation

$$\Delta E \Delta t \gtrsim 1 \quad (14)$$

holds (see, for example, [17]). In this relation ΔE is the energy uncertainty and Δt is the time interval during which the state of the system is significantly changed.

Neutrinos are detected via the observation of CC and NC reactions. In such reactions, flavor neutrinos $\nu_{l'}$, which are described by mixed coherent states (10), are detected. From (10) and (13) we find

$$|\nu_l\rangle_t = \sum_{l'} |\nu_{l'}\rangle \sum_k U_{l'k} e^{-iE_k t} U_{lk}^*. \quad (15)$$

Thus, the transition probability $\nu_l \rightarrow \nu_{l'}$ is given by

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \sum_{k=1}^3 U_{l'k} e^{-i(E_k - E_l)t} U_{lk}^* \right|^2. \quad (16)$$

From this expression it is obvious that in the case of equal energies of the neutrinos with different masses $P(\nu_l \rightarrow \nu_{l'}) = \delta_{ll'}$. *Thus, in the approach based on the Schrödinger evolution equation, there will be no neutrino oscillations if $E_k = E_l$.*

Let us assume now that the flavor neutrino states $|\nu_l\rangle$ are superpositions of the neutrino states ν_k with the same momentum \vec{p} . In this case we have

$$E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2E}, \quad (17)$$

with E being the neutrino energy at $m_k \rightarrow 0$.

Taking into account that for ultrarelativistic neutrinos

$$t \simeq L \quad (18)$$

we obtain from (16) the standard expression (5) for the transition probability which perfectly describe existing neutrino oscillation data. The time-energy uncertainty relation (14) takes the form of the well-known condition for the observation of neutrino oscillations:

$$(E_k - E_l)t \simeq \frac{\Delta m_{lk}^2}{2E} L \gtrsim 1. \quad (19)$$

Let us stress again that in the approach based on the Schrödinger evolution equation, oscillations between different flavor neutrinos are due to the fact that the neutrino state $|\nu_l\rangle_t$ is a superposition of states with *different* energies.³

³ We assumed that the states of flavor neutrinos are superpositions of states of neutrinos with different masses and the same momentum. Let us notice that if we assume that neutrinos with different masses have different momenta, in the expression for the transition probability we will have terms $(p_k - p_l)L$ in addition to the standard phases $\frac{\Delta m_{lk}^2}{2E} L$. These additional terms could be of the same order as the standard phases and could be different in different experiments. All analyses of neutrino oscillation data do not favor such a possibility.

II. Evolution in time and space.

It has been suggested in several papers (see [18, 19, 20]) that the mixed neutrino state at the space-time point $x = (t, \vec{x})$ is given by

$$|\nu_l\rangle_x = \sum_{k=1}^3 e^{-ip_k x} U_{lk}^* |\nu_k\rangle. \quad (20)$$

Here $|\nu_k\rangle$ is the state of a neutrino with mass m_k and momentum p_k . From (20) we find

$$|\nu_l\rangle_x = e^{-ip_1 x} \sum_{l'} |\nu_{l'}\rangle \sum_{k=1}^3 U_{l'k} e^{-i(p_k - p_1)x} U_{lk}^*. \quad (21)$$

Thus,

$$P(\nu_l \rightarrow \nu_{l'}) = \left| \sum_k U_{l'k} e^{-i(p_k - p_1)x} U_{lk}^* \right|^2 \quad (22)$$

is the probability to find the flavor neutrino $\nu_{l'}$ at the point x in the case that at point $x = 0$ the mixed flavor neutrino ν_l was produced. For the phase difference we have

$$(p_k - p_1)x = (E_k - E_1)t - (p_k - p_1)L = \frac{E_k^2 - E_1^2}{E_k + E_1}t - (p_k - p_1)L, \quad (23)$$

where $\vec{p}_k = p_k \vec{k}$ and $\vec{k}\vec{x} = L$ with \vec{k} being the unit vector in the direction of the momenta.

In the framework of the evolution of the flavor states in time and space two scenarios were considered in the literature.

Scenario I. General case, $t \simeq L$.

From (23) we find for the oscillation phase

$$(p_k - p_1)x = \frac{E_k^2 - E_1^2}{E_k + E_1}t - (p_k - p_1)L \simeq (p_k - p_1)\left(\frac{p_k + p_1}{E_k + E_1}t - L\right) + \frac{\Delta m_{1k}^2}{2E}t \quad (24)$$

If we assume now that the distance and the time are connected by the relation (18), we find from (24) (up to terms linear in m_k^2) the standard expression (5) for the transition probability with the oscillation phase

$$(p_k - p_1)x \simeq \frac{\Delta m_{1k}^2}{2E}L. \quad (25)$$

This result is valid in the general case $p_i \neq p_k$. This inequality means that 1. $E_k \neq E_i$; $\vec{p}_k = \vec{p}_i$ or 2. $E_k = E_i$; $\vec{p}_k \neq \vec{p}_i$ or 3. $E_k \neq E_i$; $\vec{p}_k \neq \vec{p}_i$.

Scenario II. Stationary states.

It has been suggested in [21, 18, 19] that time is not measured in neutrino oscillations and that neutrinos with different masses have the same energies, i.e. neutrino state is stationary. Taking into account that in this case the momenta of the neutrinos ν_k and ν_i are different we will come to the standard expression (5) for the transition probability between different flavor neutrinos with the oscillation phase

$$(p_k - p_1)x = \frac{\Delta m_{1k}^2}{2E}L \quad (26)$$

Thus, in all three cases which we have considered we came to the same expression (5) for the transition probability. This means that in usual neutrino oscillation experiments it is impossible to distinguish these three cases.

Recently, a new type of neutrino experiment based on the Mössbauer effect has been proposed [13, 14]. In the next section we will discuss this proposal from the point of view that it might provide a possibility to distinguish the different assumptions on the evolution of mixed neutrino states.

3 Recoilless resonant emission and absorption of antineutrinos

In [13], an experiment has been proposed for the detection of $\bar{\nu}_e$ with energy $\simeq 18.6$ keV in recoilless resonant (Mössbauer) transitions:

$${}^3\text{H} \rightarrow {}^3\text{He} + \bar{\nu}_e; \quad \bar{\nu}_e + {}^3\text{He} \rightarrow {}^3\text{H}. \quad (27)$$

For the cross section of recoilless resonant absorption of $\bar{\nu}_e$ by ${}^3\text{He}$ the value $\sigma_R \simeq 3 \cdot 10^{-33} \text{cm}^2$ was obtained [13]. With the aim to determine the value of the parameter $\sin^2 \theta_{13}$ it was proposed to study neutrino oscillations, driven by Δm_{23}^2 , in an experiment with a baseline of ~ 10 m.

It was estimated in [13] that the uncertainty of the energy of antineutrinos is of the order

$$\Delta E \simeq 8.6 \cdot 10^{-12} \text{eV} \quad (28)$$

Let us notice that ΔE is much smaller than the difference of the energies between ν_3 and ν_2

$$(E_3 - E_2) \sim \frac{\Delta m_{23}^2}{2E} \simeq 6.5 \cdot 10^{-8} \text{eV}, \quad (29)$$

which drive neutrino oscillations in the case of the approach based on evolution in time.

The state of flavor $\bar{\nu}_e$ produced and detected in the reactions (27) is the superposition of states of neutrinos with the same energy and different momenta. Thus, in the experiment proposed in [13] neutrino oscillations will not be observed if the approach based on the Schrödinger evolution equation is correct. On the other hand, neutrino oscillations can be observed in this experiment if one of the Scenarios I or II, based on the evolution in space and time, is correct. Thus, an experiment with recoilless resonant emission and absorption of antineutrinos could have an important impact on our understanding of the physics of neutrino oscillations.

In conclusion we make the following remarks:

1. In accelerator experiments K2K [22] and MINOS [23] the time of neutrino production and neutrino detection is measured. In the K2K experiment neutrino events which satisfy the criteria

$$-0.2 \leq \left| \Delta t - \frac{L}{c} \right| \leq 1.3 \mu s \quad (30)$$

were chosen. Here $\Delta t = t_{SK} - t_{KEK}$ (t_{SK} is the time of detection of neutrino events in the Super-Kamiokande detector and t_{KEK} is the time of the production of neutrinos at KEK). Thus, neutrino oscillations are a phenomenon with a finite time difference between neutrino detection and neutrino production. According to the time-energy uncertainty relation, in neutrino oscillations ΔE must be different from zero, i.e. neutrino oscillations are a *non-stationary* process. These arguments are in favor of the approach based on evolution in time (see Section 2I) and scenario I (see Section 2II).

2. In a disappearance experiment with recoilless resonance absorption of 18.6 keV antineutrinos by ^3He a positive effect of neutrino oscillations can be observed only in the case that the parameter $\sin^2 \theta_{13}$ is not too small. At present, only an upper bound $\sin^2 \theta_{13} \leq 5 \cdot 10^{-2}$ is known from the data of the CHOOZ experiment[24]. A positive result of an oscillation experiment with recoilless resonant antineutrino absorption would allow to determine the parameter $\sin^2 \theta_{13}$ and to exclude the approach based on the evolution of the mixed neutrino states in time (see Section 2I). However, a negative result of such an experiment could

be the consequence of the smallness of the parameter $\sin^2 \theta_{13}$. Thus, in the case of a negative result of an experiment with recoilless resonant antineutrino absorption, definite conclusions on the fundamentals of neutrino oscillations can be drawn only if the parameter $\sin^2 \theta_{13}$ will be measured in future reactor (DOUBLE CHOOZ [25], Daya Bay [26]) or accelerator (T2K [27], Nova [28]) experiments.

3. Effects of small neutrino masses can not be revealed in neutrino production and neutrino detection experiments with neutrino and antineutrino energies $\gtrsim 1$ MeV. Hence, the states of flavor neutrinos ν_e, ν_μ, ν_τ and flavor antineutrinos $\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$ which are produced and detected in corresponding processes, are *mixed states*. Neutrino oscillations take place because *in the propagation of states of neutrinos with definite masses* small neutrino mass-squared differences are relevant and can be determined if L/E is large enough. We considered here different assumptions on the propagation of neutrino states (in time or in space and time). Different assumptions on the propagation of neutrino states give the same transition probabilities in the case of standard neutrino oscillation experiments. We have shown that a recently proposed Mössbauer-type neutrino experiment [13, 14] could allow to distinguish the different fundamental assumptions on the propagation of neutrinos with definite masses.

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References

- [1] Super-Kamiokande Collaboration, Y. Ashie *et al.*, Phys. Rev. Lett. **93** (2004) 101801; Phys. Rev. **D71** (2005) 11205.
- [2] SNO Collaboration, Phys. Rev. Lett. **87** (2001) 071301; **89** (2002) 011301 ; **89** (2002) 011302; **92** (2004) 181301.
- [3] KamLAND Collaboration, T.Araki *et al.*, Phys. Rev. Lett. **94** (2005) 081801; hep-ex/0406035.
- [4] B. T. Cleveland *et al.*, Astrophys. J. **496** (1998) 505.

- [5] GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. **B 447** (1999) 127 ; GNO Collaboration, M. Altmann *et al.*, Phys. Lett. **B 490** (2000) 16 ; Nucl. Phys. Proc. Suppl. **91** (2001) 44; Phys. Lett. **B 616** (2005) 174.
- [6] SAGE Collaboration, J. N. Abdurashitov *et al.*, Phys. Rev. **C 60** (1999) 055801; Nucl. Phys. Proc. Suppl. **110** (2002) 315; Nucl. Phys. Proc. Suppl. **118** (2003) 39.
- [7] Super-Kamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **86** (2001) 5651.
- [8] LSND Collaboration, A. Aguilar *et al.*, Phys. Rev. D **64** (2001) 112007; G. Drexlin, Nucl. Phys. Proc. Suppl. **118** (2003) 146.
- [9] MiniBooNE Collaboration, A.A. Aguilar-Arevalo, hep-ex/0408074; R. Tayloe *et al.*, Nucl. Phys. B (Proc. Suppl.) **118** (2003) 157; Heather L. Ray *et al.*, hep-ex/0411022.
- [10] B. Pontecorvo, J. Exptl. Theoret. Phys. **33** (1957) 549. [Sov. Phys. JETP **6** (1958) 429]; J. Exptl. Theoret. Phys. **34** (1958) 247 [Sov. Phys. JETP **7** (1958) 172].
- [11] Z. Maki, M. Nakagava and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.
- [12] S.M. Bilenky, C. Giunti and W. Grimus. Prog. Part. Nucl. Phys. **43** (1999) 1.
- [13] R.S. Raghavan, hep-ph/0601079.
- [14] W. Potzel, Phys. Scr. **T127** (2006) 85.
- [15] S. Bilenky, C. Giunti, Int. J. Mod. Phys. **A16** (2001) 3931; hep-ph/0102320.
- [16] N.N. Bogolubov and D.V. Shirkov, Introduction to the theory of quantized fields (Interscience, New York, 1980).
- [17] J.J. Sakurai, Modern Quantum Mechanics, (Addison Wesley Publishing Company, New York, 1994).
- [18] H. Lipkin, Phys. Lett. **B579** (2004) 355.

- [19] B. Kayser, hep-ph/0506165.
- [20] C. Giunti, hep-ph/0409230.
- [21] L. Stodolsky, Phys. Rev. **D58** (1998) 036006.
- [22] K2K Collaboration, M.H. Alm *et al.*, Phys. Rev. Lett. **90** (2003) 041801.
- [23] MINOS Collaboration, S. Wojcicki *et al.* Proceedings of the 11th International Workshop on Neutrino Telescopes, Venice, Italy, 22-25 Feb 2005. “Venice 2005, Neutrino telescopes” 1)
- [24] CHOOZ Collaboration, M. Apollonio *et al.*, Phys. Lett. B **466** (1999) 415; M. Apollonio *et al.*, Eur. Phys. J. C **27** (2003) 331; hep-ex/0301017.
- [25] DOUBLE CHOOZ collaboration, S.A. Dazeley *et al.* Proceedings of the 7th International Workshop on Neutrino Factories and Superbeams (NuFact 05), Frascati, Italy, 21-26 Jun 2005; hep-ex/0510060
- [26] Jun Cao, Nucl. Phys. Proc. Suppl. **156** (2006) 229.
- [27] T2K Collaboration, Y. Hayato *et al.*, Nucl. Phys. Proc. Suppl. **143** (2005) 269.
- [28] D.S. Ayres *et al.* NOvA collaboration, hep-ex/0503053